Research on single cycle supply chain problem with multiple orders in an uncertain environment

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Abstract

The traditional newsboy model always assumes that retailers can only order goods before the sales season, and can only order goods once in a sales cycle. However, with the rapid and coordinated development of the logistics industry, retailers are now able to quickly replenish goods even if they face shortages during the sales season. Therefore, taking the commodities with single cycle inventory, fixed life cycle and short sales cycle as an example, this paper studies the newsboy problem that retailers can order multiple times in a single cycle with uncertain demand. By establishing the expected profit function under decentralized management mode and centralized management mode in an uncertain environment, the optimal order quantity and expected profit under the two modes are calculated and compared. It is found that when retailers cooperate with manufacturers, the profit of the supply chain is the largest, so as to explore the importance and mechanism of cooperation. Finally, the rationality of the model is verified by some numerical experiments.

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1 Introduction

Supply chain management represents a comprehensive approach and strategy for overseeing operations, particularly in logistics planning and control, covering the entire process from suppliers to end consumers. Enhancing the supply chain within businesses can lead to improved customer satisfaction, cost efficiency, and a stronger financial position. In recent years, the rise of global manufacturing has led to widespread integration of supply chain practices in manufacturing management, solidifying it as a modern management model. Some famous international enterprises such as HP, IBM, and DELL computer company have made great achievements in supply chain practice, further reinforcing the belief that supply chain management is a crucial strategy for businesses to remain competitive in the global market of the 21st century. Therefore, it has attracted many scholars and business people to study and practice supply chain management.

A reasonable supply chain contract can ensure the effective operation of the supply chain. The Newsboy model, as a supply chain contract, has attracted more and more attention. In real life, the commodities discussed in the newsboy problem are quite common, such as milk, bread, and other foods, as well as newspapers, greeting cards, and holiday gifts. Therefore, the newsboy problem should receive considerable attention in inventory management and decision analysis. The newsboy problem has a long history. Its origin can be traced back to the bank cash flow problem solved by the economist Edgewort in 1888. Silver and Peterson [1] formally put forward the classical newsboy problem for the first time. It is a single cycle and single product inventory problem and the simplest one of the newsboy problems. Based on the single-cycle supply chain research under the newsboy model, Pasternack [2] studied the pricing decision-making problem faced by producers of a commodity with a short shelf life or demand period. Firstly, the Newsboy model was used to model the supply chain channel, in which the demand was a random variable with a known distribution. Lau and Lau [3] discussed how to determine the best retail price and order quantity when there is a random correlation between random demand and retail price, so as to obtain the maximum expected profit and maximize the probability that the profit exceeds a certain target profit. Kogan [4] established the newsboy problem model of a dynamic system considering that the demand is approximately continuous distribution and there are N parallel machines in the manufacturing system. Hsieh et al. [5] extended the traditional newsboy model, considering the asymmetric information of suppliers and retailers on market demand for products with short life cycles in the supply chain. Chen [6] found that market demand is affected not only by the promotion strength of retailers, but also by the advertising expenditure of other retailers. Ockenfels and Selten studied the relationship between inventory quantity and average demand under random demand by using "impulse balance equilibrium (IBE)" under the maximization of the expected profit of the supply chain.

The aforementioned studies operate under the assumption that demand is random, but in real life, there are many uncertain factors in the market, resulting in the lack of observation data. So, it is impossible to calculate the frequency of events and determine their probability distribution. Especially for those high-tech products with short life cycles, it is usually difficult to obtain enough information, since it is difficult to use statistical data or probability distribution to describe the change in market demand. In such situations, combining subjective predictions with market conditions and consulting experts based on their prior experience becomes crucial. To address the intensity of belief in these estimates, Liu [7] created uncertainty theory, which is widely used in uncertain planning, uncertain statistics, comprehensive evaluation and other fields. In recent years, with the expansion of research depth and breadth, its application in the supply chain has also been realized. Many scholars have studied the newsboy model with uncertain random demand. Liu studied the contract design of an uncertain information supply chain based on reliability. Shen studies the uncertain sustainable supply chain network, in which demand, cost, and capacity are regarded as uncertain variables. Because there are a large number of commodities that can only sell one cycle in real life, and there are many uncertain factors that can not be described by probability in the market, it is very necessary to use uncertainty theory to study the single-cycle newsboy problem in an uncertain environment. Moreover, with the rapid coordinated development of the logistics industry and the expansion of manufacturers' origin, compared with the traditional single-cycle newsboy problem, retailers can quickly supplement goods even if they are out of stock during the sales season. Therefore, in such a market environment, this paper studies the newsboy problem that retailers can order multiple times in a single cycle with uncertain demand. It is hoped that, on the one hand, it can provide guidance for the decision-making of suppliers and retailers. On the other hand, it can encourage suppliers and retailers to reasonably allocate resources, improve resource utilization, increase profits and better complete transactions. Therefore, the research is of practical significance.

The organizational structure of this paper is as follows. In Section 2, the classical newsboy problem and two-level supply chain are introduced. In Section 3, we make assumptions and symbolic descriptions of constructing the expected profit maximization model under decentralized management mode and centralized management mode based on uncertainty theory, and obtain the optimal order quantity, respectively. In Section 4, the total revenue of the supply chain under the two management modes is compared and analyzed, and the belief degree is analyzed. Finally, Section 5 presents numerical experiments to validate the model's rationality.

2 Preliminary

2.1 Classic newsboy problem

The classical newsboy problem is the inventory problem of the single cycle and single product. Assuming that the market demand for newspapers conforms to a certain probability distribution, newsboys buy newspapers at a specific time every morning and sell newspapers at a price higher than the cost price. When the order quantity of newspapers is greater than the market demand, the remaining newspapers can only be disposed of at a price lower than the cost price. When the order quantity of newspapers is less than the market demand, the Newsboy will be punished and pay a certain fine due to shortage. The purpose of Newsboy is to maximize his expected profit by determining the optimal order quantity.

The newsboy problem has the following four characteristics.

(1) Only a single retailer is considered, and the market demand for products faced by retailers is a random variable.

(2) Retailers can only order products once before the beginning of the sales season and cannot order again during the period.

(3) The product itself has a time limit. After the time limit, the remaining products cannot be provided for the next cycle. Therefore, if there is any remaining product after the time limit, it shall be disposed of immediately.

(4) When the products ordered by the retailer do not meet the market demand, the retailer will be given a certain shortage penalty.

2.1.1 Symbol description. p : Retail price per unit product of the retailer.

ω : Wholesale price of unit products of retailers.

v : Residual value of unit product (processing price).
q : Retailer's order quantity (decision variable).
s : Penalty for shortage of unit products.
x : Market demand for products.
π : Retailer's profit.
$F(\cdot)$: Distribution function of product market demand.
$f(\cdot)$: Density function of product market demand.
r : Expected value of product market demand.

2.1.2 *Establish model.*

(1) When the market demand x is less than or equal to the retailer's order q, i.e., $x \leq q$, the Newsboy's profit is

$$\pi = px - \omega q + v\left(q - x\right). \tag{1}$$

(2) When the market demand x is greater than the retailer's order q, i.e., $x > q$, the Newsboy's profit is

$$\pi = \left(p - \omega\right) q - s\left(x - q\right). \tag{2}$$

The expected profit of the Newsboy is

$$\begin{aligned} E\left[\pi\right] = &\int_0^q \left[px - \omega q + v\left(q - x\right)\right] f\left(x\right)dx \\ &+ \int_q^\infty \left[\left(p - \omega\right) q - s\left(x - q\right)\right] f\left(x\right)dx, \end{aligned} \tag{3}$$

Equation 3) is the classical function of the classical newsboy model.

Equation 3) is a convex function, so taking the derivative of q and let $\frac{\mathrm{dE}[\Pi]}{\mathrm{dx}} = 0$ yields

$$F\left(q^*\right) = \frac{p + s - \omega}{p + s - v}, \tag{4}$$

Equation 4) is a very famous critical quantile formula in the newsboy model.

2.2 Two-level supply chain

In the current business environment, due to limited resources, the enterprise can not independently arrange all value activities. Even if all of them are done, it can not effectively obtain cost advantages due to the scale of individual enterprises. Therefore, a complete enterprise alliance can be completed between upstream and downstream enterprises through partnership.

Maloni and Benton [8] reviewed the papers on the partnership between related enterprises and divided the benefits obtained by establishing the partnership between upstream and downstream into five items.

(1) Reduction of uncertainty in supply chain management

For the buyer, it will reduce the uncertainty of factors such as cost, quantity discount, quality, and time. For the seller, it will reduce the uncertainty of market and customer demand. For both parties, it will converge objectives and consensus, reduce the impact of the external environment, and reduce speculation.

(2) Cost savings

It includes achieving the economic scale of ordering, production, and transportation, reducing management costs, integrating technical and practical processes, and improving asset utilization.

(3) Cooperative development of products and processes

Compared with upstream enterprises, downstream enterprises have more intuitive contact with the market, have a clearer understanding of market needs and hobbies, and finally promote upstream and downstream cooperation in product development.

(4) Improvement of communication

In competition with a large number of suppliers, manufacturers improve their bargaining power through negotiation.

(5) Shared risks and rewards

Enterprises unite to share information, technology, and planning work, reducing the risk caused by market uncertainty. Finally, it improves the income of partners.

Therefore, in the current business environment, it would be unwise for members of the supply chain to fight alone to improve their profits. Therefore, in order to make the whole supply system more effective and valuable, it is necessary to improve the competitive position of the enterprise, reduce costs and risks, and make the management of the whole supply system more efficient through the cooperative relationship between upstream and downstream manufacturers.

3 Uncertain single-cycle supply chain model

Due to a large number of uncertain factors in the market, we are often unable to obtain observation data, so we can neither calculate the frequency of events nor determine the probability distribution. In this case, we often need to estimate the reliability of possible events based on the experience and knowledge of experts. Therefore, in this section, based on the uncertainty theory, we introduce uncertain variables on the basis of the traditional newsboy model to find the optimal order quantity in the uncertain environment.

3.1 Basic assumptions of the model

In order to ensure that the constructed model is meaningful and simplify the complexity of calculation, we need to make some basic assumptions about the model. The mathematical model assumptions discussed in this section are as follows.

(1) Discuss the inventory of a single commodity in the cycle time.

(2) The commodity itself has timeliness, and the inventory at the end of the period cannot be transferred to the next cycle.

(3) Consumer demand for goods is a continuous uncertain variable.

(4) Whether retailers or manufacturers, they reduce the price of the remaining goods at the end of the quarter.

(5) When the retailer's goods are insufficient, it can transport the goods from the manufacturer in time, but it needs to pay additional expedited transportation expenses.

3.2 Symbol description

In order to simplify the model, we need to express the variables in the model with symbols, which are described as follows.

ξ: External demand, a nonnegative uncertain variable with uncertainty distribution Φ.

α: Expert belief degree, $\mathrm{M}(\xi \leq \mathrm{q}) = \Phi(q) \geq \alpha$ and $\alpha \in [0, 1]$.

p: Unit retail price.

c: Manufacturer's unit production cost.

ω: Manufacturer's unit wholesale price.

s: Additional transportation costs due to emergency replenishment.

v: Unit residual value of defective products and remaining products at the end of the quarter.

q: Retailer's order quantity, decision variable.

q^*: Optimal order quantity of retailers under decentralized management.

q^{**}: Optimal order quantity of retailers under centralized management.

π_r: Profits of retailers.

π_m: Manufacturer's profit.

Π: Expected total profit of supply chain.

3.3 Model under decentralized management mode

Under the decentralized management mode, retailers and manufacturers are two independent individuals, each pursuing their own profit maximization. From the perspective of retailers, the model with the goal of maximizing retailers' expected profits is as follows.

$$\begin{cases} maxE\left[\pi_r\left(q\right)\right] \\ \text{s.t. } p > \omega > c > v > 0 \\ M\left(\xi \leq q\right) \geq \alpha \\ p > \omega\left(1+s\right) \\ \xi, q \geq 0. \end{cases} \tag{5}$$

Theorem 1. Given that q is the retailer's order quantity and ξ a nonnegative uncertain variable with regular uncertainty distribution Φ. Then, the expected profit maximization model under a decentralized management mode is

$$\begin{cases} \max \begin{Bmatrix} pE\left(\xi\right) + v\int_0^q \Phi\left(\beta\right)d\beta - s\omega\int_q^{+\infty}\left[1-\Phi\left(\beta\right)\right] \\ -\omega\left[q+\int_q^{+\infty}\left(1-\Phi\left(\beta\right)\right)d\beta\right] \end{Bmatrix} \\ s.t. p > \omega > c > v > 0 \\ M\left(\xi \leq q\right) \geq \alpha \\ p > \omega\left(1+s\right) \\ \xi, q \geq 0, \end{cases} \tag{6}$$

The optimal order quantity is $q^* = \Phi^{-1}(\frac{s\omega}{s\omega+\omega-v})$

Proof. When $\xi \leq q$, the sales volume is ξ, $\Phi(q) \geq \alpha \geq 0$. The retailer's profit is

$$\pi_r\left(q,\xi\right) = p\xi + \left(\mathrm{q}-\xi\right)\mathrm{v} - \omega\mathrm{q}. \tag{7}$$

When $\xi > q$, the sales volume is q, $\Phi(q) \leq \alpha \leq 1$. The retailer's profit is

$$\begin{aligned} \pi_r\left(q,\xi\right) &= p\xi - \omega q - \left(1+s\right)\omega\left(\xi-q\right) \\ &= \left(p-\omega\right)\xi - s\omega\left(\xi-q\right). \end{aligned} \tag{8}$$

Therefore, the retailer's profit is

$$\pi_{\mathrm{r}}\left(q,\xi\right) = \begin{cases} p\xi + \left(q-\xi\right)v - \omega q,\ \xi \leq q \\ \left(p-\omega\right)\xi - s\omega\left(\xi-q\right),\ \xi > q. \end{cases} \tag{9}$$

The inverse distribution of retailer profit is

$$\Psi^{-1}\left(\alpha\right) = \begin{cases} p\Phi^{-1}\left(\alpha\right) + v\left[q-\Phi^{-1}\left(\alpha\right)\right] - \omega q,\ \Phi\left(q\right) \geq \alpha \geq 0 \\ \left(p-\omega\right)\Phi^{-1}\left(\alpha\right) - s\omega\left[\Phi^{-1}\left(\alpha\right)-q\right],\ \Phi\left(q\right) \leq \alpha \leq 1. \end{cases} \tag{10}$$

Because ξ it is a continuous uncertain variable, the retailer's total expected profit function can be expressed as follows.

$$\begin{aligned} E\left[\pi_r\left(q,\xi\right)\right] &= \int_0^{\Phi(q)}\left\{p\Phi^{-1}\left(\alpha\right) + v\left[q-\Phi^{-1}\left(\alpha\right)\right] - \omega q\right\}d\alpha \\ &+ \int_{\Phi(q)}^1\left\{\left(p-\omega\right)\Phi^{-1}\left(\alpha\right) - s\omega\left[\Phi^{-1}\left(\alpha\right)-q\right]\right\}d\alpha \\ &= p\left[\int_0^{\Phi(q)}\Phi^{-1}\left(\alpha\right)d\alpha + \int_{\Phi(q)}^1\Phi^{-1}\left(\alpha\right)d\alpha\right] + v\int_0^{\Phi(q)}\left[q-\Phi^{-1}\left(\alpha\right)\right]d\alpha \\ &-s\omega\int_{\Phi(q)}^1\left[\Phi^{-1}\left(\alpha\right)-q\right]d\alpha - \omega\left[\int_0^{\Phi(q)}qd\alpha + \int_{\Phi(q)}^1\Phi^{-1}\left(\alpha\right)d\alpha\right] \end{aligned} \tag{11}$$

Since q is the order quantity of the retailer, obviously $q \geq 0$ and $1 \geq \Phi(q) \geq 0$. Therefore $\int_0^{\Phi(q)}[q-\Phi^{-1}(\alpha)]$ can be expressed by the area S_1 in the left side of Figure 1. Similarly, $\int_{\Phi(q)}^1[\Phi^{-1}(\alpha)-q]d\alpha$ can be expressed by the area S_2 in the right side of Figure 1. The above two are obtained by integrating α. If we integrate β, we can get

$$\int_0^{\Phi(q)}\left[q-\Phi^{-1}\left(\alpha\right)\right] = \int_0^q\Phi\left(\beta\right)d\beta \tag{12}$$

$$\int_{\Phi(q)}^1\left[\Phi^{-1}\left(\alpha\right)-q\right]d\alpha = \int_q^{+\infty}\left(1-\Phi\left(\beta\right)\right)d\beta \tag{13}$$

At this time, the retailer's total expected profit function is

$$\begin{aligned} E\left[\pi_r\left(q,\xi\right)\right] &= pE\left(\xi\right) + v\int_0^q\Phi\left(\beta\right)d\beta - s\omega\int_q^{+\infty}\left(1-\Phi\left(\beta\right)\right)d\beta \\ &-\omega\left[q+\int_q^{+\infty}\left(1-\Phi\left(\beta\right)\right)d\beta\right] \end{aligned} \tag{14}$$

After calculating the retailer's expected profit function, it is necessary to prove that the model has a maximum and find the corresponding optimal order quantity.

If it is known that the uncertainty distribution Φ of the uncertainty variable ξ is continuous and q is differentiable, then $\int_0^q\Phi(\beta)d\beta$ and $\int_q^{+\infty}\Phi(\beta)d\beta$ are all differentiable. Then $E[\pi_r(q,\xi)]$ can take the derivative of q.

$$\frac{\mathrm{dE}\left[\pi_r\left(q,\xi\right)\right]}{\mathrm{d}q} = s\omega - \left[\omega\left(1+s\right)-v\right]\Phi\left(q\right) = 0, \tag{15}$$

$$\Phi\left(q^*\right) = \frac{s\omega}{s\omega+\omega-v}. \tag{16}$$

Because $\Phi(x)$ is a strictly increasing function, when $q < q^*$, $\Phi(q) < \Phi(q^*)$, that is $\frac{\mathrm{dE}[\pi_r(q,\xi)]}{\mathrm{d}q} > 0$; when $q > q^*$, $\Phi(q) > \Phi(q^*)$, that is $\frac{\mathrm{d}E[\pi_r(q,\xi)]}{\mathrm{d}q} < 0$. Therefore, $\mathrm{E}[\pi_r(q,\xi)]$ is a strictly convex function of q, the retailer's profit has a maximum value.

According to Equation 16), we can get

$$q^* = \Phi^{-1}\left(\frac{s\omega}{s\omega+\omega-v}\right), \tag{17}$$

q^* can also be interpreted as the minimum order quantity ordered by the retailer to ensure that there will be no shortage under the reliability of $\frac{s\omega}{s\omega+\omega-v}$.

The proof is completed.

Therefore, the order quantity in pursuit of maximizing the retailer's profit is q^*. At this time, the total profit function of the manufacturer and retailer can be expressed as

$$\Pi\left(q^*\right) = \mathrm{E}\left[\pi_r\left(q^*\right)\right] + \mathrm{E}\left[\pi_m\left(q^*\right)\right]. \tag{18}$$

Next, we will consider how to obtain the maximum expected profit for the whole supply chain under the centralized management mode of cooperation between retailers and suppliers.

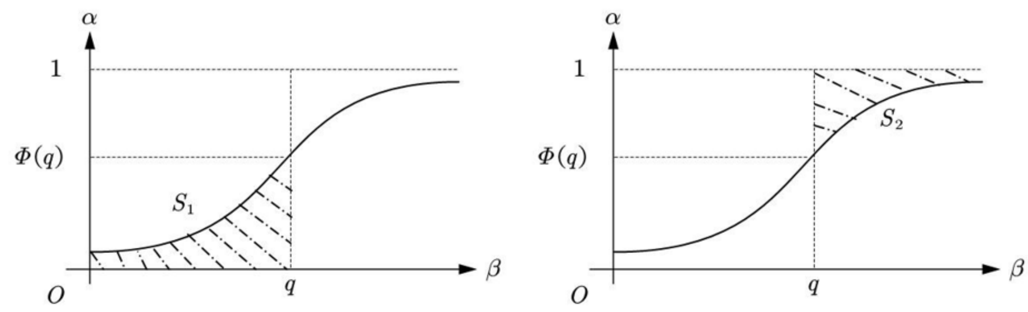

Figure 1: Transformation of proof of Theorem 1.

3.4 Model under centralized management mode

The centralized management mode can be understood as taking the manufacturer and retailer as two systems in a company. At this time, we are no longer pursuing the maximization of individual profits but the maximization of the overall interests of the company. Therefore, the model built under the centralized management mode is the expected profit maximization model of a two-level supply chain composed of retailers and manufacturers.

Theorem 2. Given that q is the retailer's order quantity and ξ a nonnegative uncertain variable with regular uncertainty distribution Φ. Then, the expected profit maximization model under a centralized management mode is

$$\begin{cases} \max\left\{\begin{array}{c} p * E(\xi) + v * \int_0^q \Phi(\beta)\,d\beta - s\omega \int_q^{+\infty}(1-\Phi(\beta))\,d\beta \\ -c\left[x + \int_q^{+\infty}(1-\Phi(\beta))\,d\beta\right] \end{array}\right\} \\ s.t. p > \omega > c > v > 0 \\ M(\xi \le q) \ge \alpha \\ p > \omega(1+s) \\ \xi, q \ge 0, \end{cases} \tag{19}$$

The optimal order quantity is $q^{**} = \Phi^{-1}(\frac{s\omega}{s\omega+c-v})$.

Proof. When $\xi \le q$, retailer sales volume is ξ, manufacturer sales volume is q, $\Phi(q) \ge \alpha \ge 0$. The manufacturer's profit is

$$\pi_m(q,\xi) = q(\omega - c). \tag{20}$$

Therefore, the profit of the supply chain is

$$\begin{aligned} \Pi(q,\xi) &= \pi_r(q,\xi) + \pi_m(q,\xi) \\ &= p\xi + v(q-\xi) - \omega q + q(\omega - c) \\ &= p\xi + v(q-\xi) - cq. \end{aligned} \tag{21}$$

When $\xi > q$, retailer sales volume is ξ, manufacturer sales volume is ξ, $\Phi(q) \le \alpha \le 1$. The manufacturer's profit is

$$\pi_m(q,\xi) = \xi(\omega - c). \tag{22}$$

Thus, the profit of the supply chain is

$$\begin{aligned} \Pi(q,\xi) &= \pi_r(q,\xi) + \pi_m(q,\xi) \\ &= (p-\omega)\xi - s\omega(\xi - q) + \xi(\omega - c) \\ &= p\xi - s\omega(\xi - q) - c\xi. \end{aligned} \tag{23}$$

So, the profit of the supply chain is

$$\Pi(q,\xi) = \begin{cases} p\xi + v(q-\xi) - cq, & \xi \le q \\ p\xi - s\omega(\xi - q) - c\xi, & \xi > q. \end{cases} \tag{24}$$

The inverse distribution of supply chain profit is

$$\Omega^{-1}(\alpha) = \begin{cases} p\Phi^{-1}(\alpha) + v\left(q - \Phi^{-1}(\alpha)\right) - cq, & \Phi(q) \ge \alpha \ge 0 \\ p\Phi^{-1}(\alpha) - s\omega\left(\Phi^{-1}(\alpha) - q\right) - c\Phi^{-1}(\alpha), & \Phi(q) \le \alpha \le 1. \end{cases} \tag{25}$$

Similarly, because ξ is a continuous uncertain variable, the retailer's total expected profit function can be expressed as follows.

$$\begin{aligned} E[\Pi(q,\xi)] &= \int_0^{\Phi(q)} \left[p\Phi^{-1}(\alpha) + v\left(q - \Phi^{-1}(\alpha)\right)\right]d\alpha \\ &\quad + \int_{\Phi(q)}^1 \left[p\Phi^{-1}(\alpha) - s\omega\left(\Phi^{-1}(\alpha) - q\right) - c\Phi^{-1}(\alpha)\right] \\ &= p\left[\int_0^{\Phi(q)} \Phi^{-1}(\alpha)\,d\alpha + \int_{\Phi(q)}^1 \Phi^{-1}(\alpha)\,d\alpha\right] + v\int_0^{\Phi(q)}\left[q - \Phi^{-1}(\alpha)\right]d\alpha \\ &\quad - s\omega\int_{\Phi(q)}^1 \left[\Phi^{-1}(\alpha) - q\right]d\alpha - c\left[\int_0^{\Phi(q)} q\,d\alpha + \int_{\Phi(q)}^1 \Phi^{-1}(\alpha)\,d\alpha\right] \\ &= p * E(\xi) + v * \int_0^q \Phi(\beta)\,d\beta - s\omega\int_q^{+\infty}(1-\Phi(\beta))\,d\beta \\ &\quad - c\left[x + \int_q^{+\infty}(1-\Phi(\beta))\,d\beta\right] \end{aligned} \tag{26}$$

According to the previous analysis, we know that $\Pi(q,\xi)$ is first-order differentiable. Next, we find the optimal order quantity under the centralized management mode.

$$\frac{\mathrm{d}E[\Pi(q,\xi)]}{\mathrm{d}q} = v\Phi(q) + s\omega - s\omega\Phi(q) - c\Phi(q) = 0, \tag{27}$$

Thus

$$\Phi(q^{**}) = \frac{s\omega}{s\omega+c-v}. \tag{28}$$

Referring to the analysis in Section 3.3, we can conclude that there is a maximum profit in the supply chain, and q^{**} is the optimal order quantity to maximize the profit in the supply chain,

$$q^{**} = \Phi^{-1}\left(\frac{s\omega}{s\omega + c - v}\right). \tag{29}$$

Therefore, when we regard the retailer and the manufacturer as two parts of a whole, the order quantity in order to maximize the profit of the supply chain is q^{**}.

The proof is completed.

At this time, the total profit function of the manufacturer and the retailer can be expressed as

$$\Pi\left(q^{**}\right) = E\left[\pi_r\left(q^{**}\right)\right] + E\left[\pi_m\left(q^{**}\right)\right]. \tag{30}$$

4 Analysis and comparison of two models

4.1 Comparison of supply chain revenue under two models

According to Section 3.3, the optimal order quantity under a decentralized management mode is q^*, and the maximum profit of the supply chain is $\Pi(q^*)$, as given by

$$q^* = \Phi^{-1}\left(\frac{s\omega}{s\omega+\omega-v}\right) \tag{31}$$

$$\Pi\left(q^*\right) = E\left[\pi_r\left(q^*\right)\right] + \pi_m\left(q^*\right) \tag{32}$$

According to Section 3.4, the optimal order quantity under a decentralized management mode is q^{**}, and the maximum profit of the supply chain is $\Pi(q^{**})$, as given by

$$q^{**} = \Phi^{-1}\left(\frac{s\omega}{s\omega+c-v}\right) \tag{33}$$

$$\Pi\left(q^{**}\right) = E\left[\pi_r\left(q^{**}\right)\right] + \pi_m\left(q^{**}\right) \tag{34}$$

Because $\omega > c$, there is $\frac{s\omega}{s\omega+c-v} > \frac{s\omega}{s\omega+\omega-v}$. Known Φ is regular, $q^{**} > q^*$ can be deduced. According to Section 3.4, we know the total revenue of the supply chain $\Pi(q)$ is a strictly convex function, and q^{**} is the optimal order quantity to maximize the expected profit. When $q^{**} > q^*$, there is $\Pi(q^{**}) > \Pi(q^*)$. In other words, the total revenue of the supply chain under the centralized management mode is greater than that under the decentralized management mode.

4.2 Analysis of belief degree

In Section 3.5, we get $\Pi(q^{**}) > \Pi(q^*)$, i.e.,

$$\begin{aligned}&p * E(\xi) + v * \int_0^{q^{**}} \Phi(\beta)\,d\beta - s\omega \int_{q^{**}}^{+\infty} (1-\Phi(\beta))\,d\beta - \\ &c\left[q^{**} + \int_{q^{**}}^{+\infty} (1-\Phi(\beta))\,d\beta\right] > \\ &p * E(\xi) + v * \int_0^{q^*} \Phi(\beta)\,d\beta - s\omega \int_{q^*}^{+\infty} (1-\Phi(\beta))\,d\beta - \\ &c\left[q^* + \int_{q^*}^{+\infty} (1-\Phi(\beta))\,d\beta\right]\end{aligned} \tag{35}$$

$$\begin{aligned}&v\left[\int_0^{q^{**}} \Phi(\beta)\,d\beta - \int_0^{q^*} \Phi(\beta)\,d\beta\right] - \\ &s\omega\left[\int_{q^{**}}^{+\infty} (1-\Phi(\beta))\,d\beta - \int_{q^*}^{+\infty} (1-\Phi(\beta))\,d\beta\right] - \\ &c\left[q^{**} - q^*\right] - c\left[\int_{q^{**}}^{+\infty} (1-\Phi(\beta))\,d\beta - \int_{q^*}^{+\infty} (1-\Phi(\beta))\,d\beta\right] > 0\end{aligned} \tag{36}$$

$$\begin{aligned}&v\int_0^{q^{**}-q^*} \Phi(\beta)\,d\beta - s\omega \int_{q^{**}-q^*}^{q} (1-\Phi(\beta))\,d\beta - c\left[q^{**} - q^*\right] - \\ &c\int_{q^{**}-q^*}^{0} (1-\Phi(\beta))\,d\beta > 0\end{aligned} \tag{37}$$

$$\begin{aligned}&v\int_0^{q^{**}-q^*} \Phi(\beta)\,d\beta + s\omega \int_0^{q^{**}-q^*} (1-\Phi(\beta))\,d\beta - c\left[q^{**} - q^*\right] + \\ &c\int_0^{q^{**}-q^*} (1-\Phi(\beta))\,d\beta > 0\end{aligned} \tag{38}$$

$$s\omega\left(q^{**} - q^*\right) > (s\omega + c - v)\int_0^{q^{**}-q^*} \Phi(\beta)\,d\beta \tag{39}$$

$$\frac{s\omega}{s\omega+c-v} > \frac{\int_0^{q^{**}-q^*} \Phi(\beta)d\beta}{q^{**}-q^*} \tag{40}$$

Since $\Phi(q^{**}) = \frac{s\omega}{s\omega+c-v}$, there is

$$\Phi\left(q^{**}\right) > \frac{\int_0^{q^{**}-q^*} \Phi(\beta)d\beta}{q^{**}-q^*} \tag{41}$$

We can explain that the retailer believes that he has a chance greater than $\frac{\int_0^{q^{**}-q^*} \Phi(\beta)d\beta}{q^{**}-q^*}$, and his order quantity is less than the optimal order quantity q^{**} in the supply chain. In other words, the probability that the retailer believes that its order quantity is greater than the optimal order quantity q^{**} of the supply chain is greater than $(1 - \frac{\int_0^{q^{**}-q^*} \Phi(\beta)d\beta}{q^{**}-q^*})$ Thus, $\frac{\int_0^{q^{**}-q^*} \Phi(\beta)d\beta}{q^{**}-q^*}$ can be interpreted as the lower bound of the reliability of "the order quantity is less than the optimal order quantity q^{**} of the supply chain".

5 Numerical experiment

5.1 Example description

A large chain bookstore sells a new monthly magazine. Relying on previous sales experience, the shopkeeper subjectively believes that the monthly market demand obeys the linear uncertain distribution (500,600). The production cost of the monthly magazine is 7. The printing factory sells it to the retailer at the price of 10 per copy, and the retailer sells it at the unit price of 15. If there is a surplus at the end of the month, the retailer will reduce the price by 5 per copy. In case of shortage, the bookstore can transport the incoming goods from the printing factory in time or buy them at a higher price from other places. At this time, the unit price cost of the monthly magazine increases by 3 additional expenses on the basis of the wholesale price. In the sales process, bookstores can choose one of decentralized management mode and centralized management mode to determine the number of books and periodicals ordered.

5.2 Comparison of two modes

From the above, we know that c = 7; ω = 10; p = 15; v = 5; s = 0.3:

When we pursue the bookstore's profit maximization under the decentralized management mode, we can get

$$\begin{aligned}\alpha^* &= \Phi\left(q^*\right) = \frac{s\omega}{s\omega+\omega-v} = \frac{0.3\times 10}{0.3\times 10+10-5} = 0.375 \\ q^* &= \Phi^{-1}\left(\alpha^*\right) = 500\left(1-\alpha\right) + 600\alpha = 537.5\end{aligned} \tag{42}$$

At this time, the belief degree is 0.375, and the optimal order quantity is 537.5. It can be understood that under the confidence level of 0.375, the shopkeeper needs to order 537.5 monthly magazines so that there will be no shortage.

When we pursue the maximum total profits of the bookstore and printing plant under the centralized management mode, we can get

$$\begin{aligned}\alpha^{**} &= \Phi\left(q^{**}\right) = \frac{s\omega}{s\omega+c-v} = \frac{0.3\times 10}{0.3\times 10+7-5} = 0.6 \\ q^{**} &= \Phi^{-1}\left(\alpha^*\right) = 500\left(1-\alpha\right) + 600\alpha = 560.\end{aligned} \tag{43}$$

In centralized management mode, the belief degree is 0.6, and the optimal order quantity is 560. It can be understood that under the belief degree of 0.6, the shopkeeper needs to order 560 monthly magazines so that there will be no shortage.

Therefore, we can see that when all parameters remain unchanged, the confidence level of the centralized management mode is greater than that of the decentralized management mode. And because uncertain distribution Φ is regular, with the greater change of the belief degree of the centralized management mode, the optimal order quantity will be greater.

Table 1: Effect of changes in s and v.

s	v	$\Phi(q^*)$	$\Phi(q^{**})$	q^*	q^{**}	$\Pi(q^*)$	$\Pi(q^{**})$	$\frac{\int_{q^*}^{q^{**}} \Phi(\beta)d\beta}{q^{**}-q^*}$
2	4	0.2500	0.4000	525.0000	540.0000	4334.3750	4340.0000	0.3250
	5	0.2857	0.5000	528.5714	550.0000	4340.8163	4350.0000	0.3929
	6	0.3333	0.6667	533.3333	566.6667	4350.0000	4366.6667	0.5000
	7	0.4000	1.0000	540.0000	600.0000	4364.0000	4400.0000	0.7000
3	4	0.3333	0.5000	533.3333	550.0000	4316.6667	4325.0000	0.4167
	5	0.3750	0.6000	537.5000	560.0000	4327.3438	4190.0000	0.4875
	6	0.4286	0.7500	542.8571	575.0000	4341.8367	4362.5000	0.5893
	7	0.5000	1.0000	550.0000	600.0000	4362.5000	4400.0000	0.7500
4	4	0.4000	0.5714	540.0000	557.1429	4304.0000	4314.2857	0.4857
	5	0.4444	0.6667	544.4444	566.6667	4318.5185	4333.3333	0.5556
	6	0.5000	0.8000	550.0000	580.0000	4337.5000	4360.0000	0.6500
	7	0.5714	1.0000	557.1429	600.0000	4363.2653	4400.0000	0.7857

5.3 Sensitivity analysis

The additional transportation cost and the residual value of the remaining products will have an impact on the retailer's order quantity and the total revenue of the supply chain. In order to explore the impact degree, we keep other parameters unchanged. Some numerical results of s = 0.2,0.3,0.4 and v = 4,5,6,7 are given in Table 1.

Firstly, we can see that the optimal order quantity q^{**} under the centralized management mode is always greater than the optimal order quantity q^* under the decentralized management mode, and the total expected profit $\Pi(q^{**})$ of the supply chain under the centralized management mode is greater than the total expected profit $\Pi(q^*)$ of the supply chain under the decentralized management mode.

When v remains unchanged, $\Pi(q^{**})$ and $\Pi(q^{**})$decrease with the increase of s. When s remains unchanged, $\Pi(q^{**})$ and $\Pi(q^{**})$ increase with the increase of v.

When v = 7, we can see $\Phi(q^{**}) == 1$ and $\Pi(q^{**}) == 4400$. This means that when the residual value of the monthly magazine is high and can be handled, retailers in the centralized management mode 100% believe that ordering 600 books will not be out of stock, and the supply chain revenue will reach the maximum at this time.

Finally, we can see that $\frac{\int_{q^*}^{q^{**}} \Phi(\beta)d\beta}{q^{**}-q^*}$ is always less than $\Phi(q^{**})$. When other parameters remain unchanged, whether s or v increases, $\frac{\int_{q^*}^{q^{**}} \Phi(\beta)d\beta}{q^{**}-q^*}$ will increase accordingly. This means that the increase of additional transportation fee or commodity salvage value will increase the shopkeeper's belief degree that the market demand is less than the optimal order quantity q^{**}.

6 Conclusions

Aiming at many uncertain factors in the market, this paper studies the single-cycle supply chain problem that can be ordered multiple times in an uncertain environment. Regarding market demand as an uncertain variable, we explore, compare, and analyze the optimal order quantity and total revenue of the supply chain under decentralized management mode and centralized management mode, and we finally use a numerical example to verify the rationality of the model. On the one hand, it helps businesses make decisions to maximize the expected profits when selling new products without historical data. On the other hand, we can see that the overall benefits of the supply chain can be maximized when upstream and downstream enterprises cooperate, so the cooperation between enterprises is very necessary.

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